

Table of Contents

Preface	5
Table of Contents	7
 I Coursebook 3: Integer Polynomials and Arithmetic Methods	 9
1 Algebraic Systems and Equation Methods	11
1.1 Piecewise & Discrete Equations: Floors, Ceilings, Absolute Values, Casework.....	12
1.1.1 Solved Examples: Floors, Ceilings, and Absolute Values	12
1.1.2 Exercises: Floors, Ceilings, and Absolute Values.....	25
1.2 Arithmetic of Polynomial Roots: Power Sums, Transformations, Root Relations	29
1.2.1 Further Problems on Polynomial Roots	29
1.2.2 Power Sums of Roots: Newton's Identities & Linear Recurrences	52
1.2.3 Constructing Polynomials from Roots: Reverse Viète & Transformations	60
1.2.4 Common Roots: Elimination, Resultants, Parameter Conditions	81
1.3 Mixed Systems of Equations: Symmetry, Substitution, Olympiad Casework	96
1.3.1 Solved Examples: Mixed Systems	96
1.3.2 Exercises: Mixed Systems	114
2 Arithmetic Methods for Polynomial Problems	123
2.1 Divisibility, Orders, and Lifting	125
2.1.1 Arithmetic Definitions and Theorems.....	125
2.1.2 Solved Examples: Arithmetic Methods	129
2.2 Polynomials Modulo an Integer	137
2.2.1 Modular Polynomial Theorems	137
2.2.2 Lagrange's Lemma and The Most Useful Number Theoretic Polynomial	139
2.2.3 Legendre's Formula, Digit-sum Form, and p -adic Power Sums	144
2.2.4 Lucas and Kummer in a Freshman's Dream	146
2.2.5 Advanced Analytic Theorems on Polynomials	150
2.3 Number-Theoretic Polynomial Problems.....	153
2.3.1 Solved Examples: Number-Theoretic Polynomials	153
2.3.2 Exercises: Number-Theoretic Polynomials	163
3 Selected Polynomial Problems	171
3.1 Integer Polynomials, Finite Differences, and Irreducibility	172
3.1.1 Polynomial Structure, Division, and Identity	172
3.1.2 Integer Polynomials, Finite Fields, and Discrete Bases	177
3.1.3 Composition, Symmetry, and Algebraic Structure	184
3.1.4 Root Data, Multiplicity, and Equality	186
3.1.5 Complex, Fourier, and Elimination Methods	191
3.1.6 Reversible Transformations and Constructions	193
3.2 Integer, Rational, or Number-Theoretic Constraints	196

3.3	Irreducibility, Factorization, and Decomposition	212
3.4	Summation and Finite Difference Identities.....	222

II Complete Solutions for Coursebook 3 **231**

1 Algebraic Systems and Equation Methods **233**

1.1	Piecewise & Discrete Equations: Floors, Ceilings, Absolute Values, Casework.....	233
1.2	Arithmetic of Polynomial Roots: Power Sums, Transformations, Root Relations	252
1.3	Mixed Systems of Equations: Symmetry, Substitution, Olympiad Casework	263

2 Arithmetic Methods for Polynomial Problems **295**

2.1	Number-Theoretic Polynomial Problems.....	295
-----	---	-----

3 Selected Polynomial Problems **331**

3.1	Summation and Finite Difference Identities.....	331
-----	---	-----

Bibliography **333**

Physical Resources.....	333
-------------------------	-----

Online Resources	333
------------------------	-----

Acronyms.....	335
---------------	-----

Part I: List of Problems **337**

Part II: List of Problems and Solutions **341**

List of Theorems, Definitions, and Identities **343**

Indexes **344**

Index of Concepts.....	344
------------------------	-----

Index of Methods	345
------------------------	-----

Index of Theorems	347
-------------------------	-----

Index of People.....	348
----------------------	-----

Index of Tools & Problem Types	349
--------------------------------------	-----

Index of Sources	350
------------------------	-----