

Table of Contents

Preface	5
Table of Contents	6
 I Lyran Trigonometry Coursebook 4: Triangle Trigonometry	 9
1 Half-Angle and Double-Angle Lessons	11
1.1 Doubling and Halving Identities	11
1.2 Half- and Double-Angle Exercises	17
2 Triangle Laws and Geometric Quantities	25
2.1 Triangle Laws and Projection Problems	25
2.2 Perimeter and Area Problems	31
2.3 Heights and Orthic Problems	37
2.4 Internal and External Angle-Bisector Problems	46
2.5 Inradius and Exradius Problems	53
2.6 Triangle Centre Distance Problems	65
2.7 Contemporary Applications of Triangle Trigonometry	74
2.7.1 Right-Triangle, Projection, and Metric Setups	74
2.7.2 Angle Bisectors, Half-Angles, and Special-Angle Determination	78
2.7.3 Incircles, Excircles, and Arc Midpoints	84
2.7.4 Orthocentres, Orthic Triangles, and Altitude Configurations	88
2.7.5 Advanced Triangle-Circle Methods: Inversion, Ratio Lemmas, and Radical Axes	96
3 Quadrilateral Trigonometry	107
3.1 Universal Diagonal and Area Identities	107
3.2 Cyclic Quadrilaterals	110
3.3 Tangential and Bicentric Quadrilaterals	112
3.4 Quadrilateral Contest Applications	114
 II Complete Solutions for Coursebook 4	 121
Half-Angle and Double-Angle Lessons	123
Half- and Double-Angle Exercises	123
Triangle Laws and Geometric Quantities	135
Triangle Laws and Projection Problems	135
Perimeter and Area Problems	148
Heights and Orthic Problems	160

Internal and External Angle-Bisector Problems	169
Inradius and Exradius Problems	186
Triangle Centre Distance Problems	205
Contemporary Applications of Triangle Trigonometry	220
Quadrilateral Trigonometry	243
Universal Diagonal and Area Identities	243
Cyclic Quadrilaterals	246
Tangential and Bicentric Quadrilaterals	251
Quadrilateral Contest Applications	256
Bibliography	263
Physical Resources	263
Online Resources	263
Acronyms	264
Notation	264
List of Problems in Part I	265
List of Solved Problems in Part II	267
List of Theorems, Definitions, and Identities	268
List of Figures	269
Indexes	270
Index of Concepts	270
Index of Methods	270
Index of Theorems	271
Index of People	272
Index of Tools & Problem Types	272
Index of Sources	273